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The capture of mesons

Part II

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Introduction

In a preceding paper [1][1] the process $\mu^- + P \rightarrow N$ was re-examined and calculations were carried out on an α-particle model of the nucleus. In agreement with earlier results [7], it could be concluded that in the case of indirect capture with the meson rapidly falling down into a K-orbit (s-orbit) CONVERSI, PANCINI and PICCIONI's experiments [4] could not be accounted for on the assumption that the μ-mesons are responsible for a substantial part of the nuclear forces. Because of the possible errors in the calculations it seemed, on the other hand, not out of the question in the case of direct capture — at least with scalar mesons — to achieve agreement with experimental results, given the same assumption. This way out of the difficulties was not, however, taken too seriously, since both theoretical considerations concerning the slowing down of mesons and the experimentally established law governing the variation of the capture probability with atomic number seemed to favour the view that the meson is captured indirectly. Recent investigations [8, 12, 21] have supported this view, and the question may now be considered as settled.

We must in all probability rule out the possibility that the time needed for the meson to fall down from an orbit with high angular momentum into an s-orbit will determine the lifetime for capture. As has been pointed out by OCCHIALINI and POWELL [17], it is then difficult to understand the fact that negative π-mesons are captured by both light and heavy nuclei, although the lifetime for μ-decay is about four hundred times smaller than that of the μ-mesons for β-decay. Estimates of the time needed for falling down into an s-orbit also argue against this solution [8; unpublished calculations by FERRETTI and by BETHE].

As already pointed out in Part I, one should, on account of the strong interaction between the nucleons, expect a star at the end of most meson tracks, if the process $\mu^- + P \rightarrow N$ were the dominating one. This does not agree with observations. Since the process, just mentioned, is supposed to be more probable than the alternative processes, if the μ-mesons interact directly with the nucleons, the results hint again at an indirect interaction, probably

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[1] In the following reference will be made to this as Part I and to formulae in it by putting a I before the number of the formula in question.

via the heavy mesons, the π-mesons. Calculations on such a model — with a capture resulting in a neutral meson — have been carried out by LODGE [15]. The agreement with experiments is good, considering the uncertainty of the estimates.

Thus the calculations in Part I are most likely not applicable to the μ-mesons. Instead, we may use them for π-mesons. The work carried out in BRISTOL [17] and BERKELEY [9, 16] give evidence of stars, where negative π-mesons end in photographic emulsions. We may therefore admit the process

$$\pi^- + P \rightarrow N \tag{1}$$

rather than

$$\pi^- + P \rightarrow N + h\nu \tag{2}$$

or

$$\pi^- + P \rightarrow N + \pi^0 \tag{3}$$

since the two latter processes will usually give rise to a fairly small excitation of the nucleus with a surplus of neutrons.

The modifications of the results obtained in Part I when applied to scheme (1) will be discussed in the following. In this connection some further remarks about the calculations in Part I are made.

The possible considerable errors in the evaluation of the probability of process (1) make a comparison with process (2) desirable, since this, as is known from earlier calculations [2, 3, 10, 13], is likely to be only slightly less probable than the former.[1] Unlike process (1), process (2) is possible for a free proton and can, therefore, be treated with greater accuracy than the latter. The binding of the capturing nucleon cannot, however, even here be neglected, as will be shown in the following, but will — except for the case of scalar mesons — cause a considerable reduction in the probability. The influence of the COULOMB interaction between the meson and the nucleons will also be discussed.

The process $\pi^- + P \rightarrow N$

When applying the results of Part I to scheme (1) the necessary modifications arising from the different masses of the π- and μ-mesons are easily carried out. Another point needs, however, more attention. In Part I the capture probability was calculated for an α-particle, which is transformed by the capture into a triton nucleus (in its ground state) and a free neutron. The inclusion of other possible final states was assumed not to alter the cross sections essentially. With increasing mass of the meson the added contribution from processes leading to a total disruption of the α-particle will gain in importance, since the increase in the number of possible states will partly compensate the decrease in the contribution from each state separately.

Because of this and in order to investigate the validity of the special assumption in Part I, the probability of a capture resulting in a total disruption of the α-particle has been computed in the Appendix.[1] The calculations refer to a vector meson and the nucleons are treated as free in the final state. The capture probability, evaluated for a meson with a rest energy of 100 MeV, is found to be about 7 times greater than the value obtained in Part I (cf. I: 114). The use of other meson theories will not alter the result essentially.

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[1] If the mass of the π^0-meson is not too small compared with that of the π^--meson, process (3) will, on the other hand, be much less probable than process (1) and (2), when a slow meson is captured.

The treatment of the nucleons as free in the final states implies, however, a very crude approximation, because the only part of the wave function of the nucleons entering into the calculations is that describing the nucleons when all clustered together within a small volume of the order of magnitude of that of the α-particle. The interaction between the nucleons cannot then be neglected. It will bring about a more rapid oscillation of the interesting part of the wave function. The destruction by interference, already prominent before, will thereby gain in importance causing a reduction in the capture probability.

A rough idea of the order of magnitude of the correction to be expected may be obtained in the following way. After the capture the mean value of the kinetic energy of each of the nucleons when widely separated is about 18 MeV. At small distances the kinetic energy is increased by the depth of the effective potential for the nucleon. In the region under consideration a doubling of the kinetic energy is by no means unreasonable. The corresponding modification in the capture probability, as given by (60) in the Appendix, results in a correction factor of about $\frac{1}{200}$[2], thus bringing the probability down below that of a final state with a triton.[3, 4] Because of the uncertainty of the estimate of the influence of the interactions in the final state the argument is not, however, conclusive.

Returning to the modifications necessary for the application of the results to a π-meson, the following can now be said. The mass of the π-meson, m_π, will be assumed to be 285 electron masses (rest energy $= 146$ MeV). If a final state with a triton nucleus dominates, when the π-meson is captured, the passing over from a 100 MeV meson to a 146 MeV meson will — in the case of a pseudoscalar or vector meson — bring about a change in the capture probability by a factor of about $\frac{1}{20}$, as an easy calculation shows. If again the other possible final states make the decisive contribution already for a 100 MeV meson, the corresponding factor is about $\frac{1}{6}$. The two values give the limits for the change.

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[1] Final states with a deuteron and two neutrons are also possible, but the contribution will be comparatively small because of the large radius (small binding energy) of the deuteron. No information seems so far to have been obtained about the excited states of the triton. If any, they are few and will not substantially affect our results.

[2] It is interesting that a rough estimate of this kind gives a correction factor fairly independent of, or even slightly increasing with, the energy of the captured meson. This is caused by the choice of the error function for description of the α-particle. A wave function of the hydrogen type will, on the other hand, give a correction factor decreasing with the energy, which is of course what we would be inclined to expect.

[3] The influence of the interaction between the triton and the emitted neutron is, as has already been pointed out in Part I, of minor importance, firstly because the triton is saturated with neutrons and secondly because the correction refers to only *one* emitted nucleon.

[4] In the ordinary photo-effect a decrease in the cross section is consistently caused by taking the COULOMB interaction between the emitted electron and the nucleus into consideration. In the neighbourhood of the absorption edge, where we have the greatest effect, the cross section for photoabsorption by a K-electron is diminished by a factor $\frac{1}{5}$ à $\frac{1}{10}$, when calculated using the exact wave functions. Considering that the integration involved in the calculations refers to one set of relative coordinates (x_1, x_2, x_3) instead of three independent sets of relative coordinates (e. g. $\boldsymbol{r}^{(12)}, \boldsymbol{r}^{(13)}, \boldsymbol{r}^{(14)}$) in our case, the comparatively small effect is understood. Taking over the result from the photo-effect, we should in our case expect a correction factor of the order of magnitude 10^{-2} à 10^{-3}, agreeing fairly well with the value obtained above. This procedure is, however, of course very rough.

The process $\pi^- + P \rightarrow N + h\nu$

General remarks. In earlier calculations on this process the meson and the proton were treated as free. In the case of mesons with low energy, the only one considered here, it is of course evident that the COULOMB attraction between the capturing nucleus and the negative meson will increase the capture probability because of the increased probability of finding the meson near a proton. One should expect to be able to correct roughly for this by multiplying by the square of the wave amplitude of the meson inside the nucleus. That this is so in one-step transitions is immediately seen. On the other hand, in two-step transitions, which also play an important rôle, it is not so clear how this is brought about. The main contribution here comes from two alternative processes:

(i) The meson emits the photon and is then absorbed by the proton.

(ii) The proton emits a positive meson, which then annihilates the negative meson under emission of the photon.

In neither of the processes is the meson absorbed in its original state. In contrast to the free case, the intermediate state in (i) or (ii) is not, however, fixed. Because of the indeterminate momentum of the original meson, several ways are possible. The interference of all these gives in the end the probability roughly to be expected.

General arguments concerning the influence of the nuclear bindings can be obtained in the following way. Let

$$\triangle E \approx m_\pi c^2 \tag{4}$$

represent the energy converted in the capture process. During the time $\triangle t$ needed for the event, the localization of $\triangle E$ is undetermined. We can therefore use the time part of HEISENBERG's indeterminacy relation

$$\triangle E \cdot \triangle t \gtrsim \hbar. \tag{5}$$

Influence upon the process is consequently to be expected from a sphere with the capturing nucleon at its centre and with radius

$$r \approx c \triangle t \gtrsim \frac{\hbar}{m_\pi c} = 1.35 \cdot 10^{-13} \text{ cm}. \tag{6}$$

Thus r is at least of the same order of magnitude as the COMPTON wave length of the meson. This is of the order of magnitude of the distance to the neighbouring nucleons, which must, therefore, be expected to affect the process. Since the influence from more distant nucleons, on the other hand, can be assumed to be relatively small, one is — except for the lightest nuclei — naturally led to base the calculations on an α-particle model of the nucleus, as was in fact done in Part I.

The discussion is equally applicable to the capture and to the production of mesons with low energy. With increasing energy of the meson the binding loses in importance, as is seen from (5) and (6). In the case of processes like (1), where the binding is necessary for the fulfilment of the conservation laws, this results in a rapidly diminishing probability of the occurrence of the process. In considering other processes the nucleons can be treated as free with better and better approximation.

On the other hand, in processes involving hypothetical mesons with small mass and low kinetic energy the influence from more distant nucleons cannot be neglected and it is imperative to treat the nucleus as a whole.

The qualitative arguments will be confirmed and a quantitative estimate of the effect of the binding will be obtained from the calculations below.

Calculations for pseudoscalar mesons. We shall use the notations in Part I and assume that the events take place in a periodicity cube of unit volume. Let $A_\mu\,(\boldsymbol{A},\, i A_0)$ be the electromagnetic four-potential. When introducing the coupling between the nuclear and meson fields on one side and the electromagnetic field on the other, we must — in order to secure the gauge invariance — make the substitutions

$$\frac{\partial}{\partial x_\mu} \rightarrow \frac{\partial}{\partial x_\mu} - \frac{i e}{\hbar c} A_\mu \tag{7}$$

everywhere in the Lagrangian describing the nuclear and meson field and their interaction. The corresponding modification of the Hamiltonian is then easily obtained. We thus get the following coupling terms (cf. Part I, p. 21, 22)

$$\text{(8 a)}\quad H_{MN} = c\sqrt{4\pi}\, g_{PS}\, \bar{\omega}\, \bar{U} + \frac{c\sqrt{4\pi}\, f_{PS}}{\varkappa} \left\{ (\overline{\mathbf{S}} \cdot \nabla)\, \bar{U} + \frac{\bar{S}_0}{c} P^*_{\bar{U}} \right\} + \\ + \text{conjugate complex.}$$

$$\text{(8 b)}\quad H_{AM} = \frac{i e c}{\hbar} \{ \bar{U}^* (\boldsymbol{A} \cdot \nabla \bar{U}) - (\nabla \bar{U}^* \cdot \boldsymbol{A})\, \bar{U} \} + \frac{e^2}{\hbar^2} A^2\, \bar{U}^*\, \bar{U} - \\ - \frac{i e}{\hbar} A_0 \,(P_{\bar{U}}\, \bar{U} - \bar{U}^*\, P^*_{\bar{U}}).$$

$$\text{(8 c)}\quad H_{AMN} = -\frac{i e \sqrt{4\pi}\, f_{PS}}{\hbar \varkappa} (\overline{\mathbf{S}} \cdot \boldsymbol{A})\, \bar{U} + \text{conjugate complex)} \qquad \left(\varkappa = \frac{m_\pi c}{\hbar} \right)$$

where the indices A, M and N refer to the electromagnetic, meson and nuclear fields, respectively. The term H_{AN} has been omitted, since it is unimportant for our purposes.

In order to use perturbation theory, the unperturbed Hamiltonian must be brought into diagonal form. This will not be the case if we decompose $\bar{U}$ and $P_{\bar{U}}$ in plane waves (cf. I: 52), since we must include the Coulomb interaction between the meson and the nucleons in the unperturbed term. As the mass of the meson is considerably smaller than that of the nucleus, we shall treat the electric field, acting on the meson, as static. In doing this, we have in addition neglected the variations in the electric field inside the nucleus due to the internal motion; a smoothing out of the field is thus presupposed. We can now apply the results of SNYDER and WEINBERG [18; cf. also 6], who solved the problem of diagonalizing in the presence of a static electric field. Modifying their manner of writing we put

$$(9)\quad \left\{\begin{aligned} \bar{U} &= \sum_p \frac{\hbar}{\sqrt{2E_p}} a_p u_p + \sum_n \frac{\hbar}{\sqrt{2E_n^L}} b_n^* (u_n^L)^* \\ P_{\bar{U}} &= \sum_p \frac{1}{\sqrt{2E_p}} i a_p^* (E_p - eV) u_p^* - \sum_n \frac{1}{\sqrt{2E_n^L}} i b_n (E_n^L + eV) u_n^L. \end{aligned}\right.$$

Here V is the electric scalar potential. a_p, a_p^* and b_n, b_n^* are creation and annihilation operators for positive and negative mesons, respectively. u_p satisfies the KLEIN-GORDON equation

$$(10\text{ a})\qquad \{(E_p - eV)^2 + \hbar^2 c^2 \triangle - m_\pi^2 c^4\}\, u_p = 0$$

with E_p positive, while u_n^L satisfies the charge-conjugated wave equation (cf. [14])

$$(10\text{ b})\qquad \{(E_n^L + eV)^2 + \hbar^2 c^2 \triangle - m_\pi^2 c^4\}\, u_n^L = 0$$

for positive E_n^L. Thus u_n^L is an eigensolution of (10 a) belonging to the negative energy eigenvalue $-E_n^L$. The wave functions u_p and u_n^L do not build up an orthonormal set of wave functions but satisfy the relations[1]

$$(11)\quad \left\{\begin{aligned} &\int u_p^* (E_p + E_{p'} - 2eV)\, u_{p'} = 2E_p\, \delta_{pp'} \\ &\int (u_n^L)^* (E_n^L + E_{n'}^L + 2eV)\, u_{n'}^L = 2E_n^L\, \delta_{nn'} \\ &\int u_p^* (E_p - E_n^L - 2eV)\, u_n^L = 0. \end{aligned}\right.$$

As in Part I, we shall perform the calculations in the configuration space of the nucleons, while the electromagnetic and meson fields are still quantized. The perturbation H_1, arising from the coupling terms (8), can now in the case of n nucleons be written

$$(12)\qquad H_1 = \sum_{j=1}^{n} H_1^{(j)} + \int H_{AM}\, d\tau^{(OT)}$$

with

$$(13)\qquad H_1^{(j)} = H'_{MN}(\boldsymbol{r}^{(j)}) + H'_{AMN}(\boldsymbol{r}^{(j)})$$

where $d\tau^{(OT)}$ is the volume element of the coordinates $\boldsymbol{r}^{(OT)}$ relative to the centre of gravity $\boldsymbol{r}^{(T)}$ of the nucleus, considered approximately at rest. $H'_{AMN}(\boldsymbol{r}^{(j)})$ is obtained in the same way as $H'_{MN}(\boldsymbol{r}^{(j)})$, which is given by (I: 9, 10).

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[1] Because of the weakness of the electric field the conditions (11) can always be fulfilled.

The matrix element $H_{1,IF}$ for the transition in question can be written

$$H_{1,IF} = H^{\text{direct}}_{1,IF} + \sum_{(i)} \frac{H_{1,I(i)} H_{1,(i)F}}{E_I - E_{(i)}} + \sum_{(ii)} \frac{H_{1,I(ii)} H_{1,(ii)F}}{E_I - E_{(ii)}} \tag{14}$$

where the indices (i) and (ii) refer to the two-step processes considered on p. 22. The probability of capture for the mesons with low kinetic energy studied here is considered as approximately independent of the direction and polarization of the emitted light quantum of energy $h\nu$. If, however, the capturing nucleus has a spin differing from zero, this is not strictly true but can be provided for by averaging over the possible spin directions of the nucleus, as is always tacitly understood in the following. For the capture probability P^{ph}_{PS} per proton in a nucleus with n_p protons we thus get

$$P^{ph}_{PS} = \frac{2\pi}{\hbar n_p} \sum_F \varrho_{ph} \, | H_{1,IF} |^2. \tag{15}$$

The summation has to be carried out over all final states of the nucleons, while the number of final states ϱ_{ph} for the photon per unit energy interval is — neglecting the recoil energy — given by

$$\varrho_{ph} = 8\pi \frac{(h\nu)^2}{h^3 c^3}.^1 \tag{16}$$

As already pointed out, the main contribution to (15) comes from states of the nucleus with fairly small excitation energy (≈ 10 MeV).[2] We thus have

$$h\nu \approx m_\pi c^2 - 10 \text{ MeV} \approx 135 \text{ MeV} \quad \text{and} \quad \frac{\lambda}{2\pi} = \frac{c}{2\pi\nu} \approx 1.5 \cdot 10^{-13} \text{ cm} \tag{17}$$

and ϱ_{ph} can roughly be considered as a constant and taken outside the summation in (15).

In order to avoid complications we shall, moreover, confine ourselves to the case where the wave function of the original meson is roughly a constant inside the nucleus. This excludes nuclei with $n_p > 30$ or so. It must be remarked that the strict solution of the relativistic wave equations (10) in the case of an $\frac{1}{r}$-potential approaches infinity slowly at the origin for an s-state (cf. [19]). If the ordinary perturbation theory is applied, one would get the result that a meson in an s-state of a hydrogen nucleus should be captured in no time. There seems, however, to be reason to believe that the potential has to be modified at small distances from the centre of action, so that we get a solution which is finite at the origin.

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[1] Here a factor 2 arises from the two possible polarizations of the photon.

[2] For light nuclei, especially, the capture may, of course, result in the emission of one or more nucleons of low energy in addition to the emitted photon with the residual nucleus in its ground state or in an excited state.

Using the perturbation theory for a many-body problem, outlined in Part I, we can compute the transition element (14). Considering that the energy of the original meson is approximately $m_\pi c^2$, we thus get

$$(18)\quad \begin{aligned} H^{\text{direct}}_{1,IF} = \sum_{j=1}^{n} H'_{AMN,IF}(\boldsymbol{r}^{(j)}) &= -\frac{4\pi i e f_{PS}}{\hbar\varkappa}\cdot\frac{\hbar c}{\sqrt{2h\nu}}\cdot\frac{\hbar}{\sqrt{2m_\pi c^2}}\cdot \\ &\cdot\sum_{j=1}^{n}\left\{\int u_I^L(\boldsymbol{r}^{(jT)})^*\cdot e^{\frac{i}{\hbar}\boldsymbol{k}\cdot\boldsymbol{r}^{(jT)}}\cdot\varphi^*_{I,0}\,\sigma_e^{(j)}\,T_N^{(j)}\,\varphi_{F,0}\cdot d\tau^{(1l)}\right\} \end{aligned}$$

where u_I^L is the wave function of the original meson and $\varphi_{I,0}$ and $\varphi_{F,0}$ the wave functions of the relative motion of the nucleons in the zero approximation (cf. I: 32) for the initial and final states, respectively. $\sigma_e^{(j)}$ is the projection of $\sigma^{(j)}$ in the direction of polarization $\boldsymbol{e}$ of the emitted light quantum with momentum $\boldsymbol{k}$.

$H_{1,I(i)}$ is the matrix element for the emission of the photon by the meson, which jumps into a state $u^L_{(i)}$. Neglecting the kinetic energy of the original meson, we find by means of (8 b)

$$(19)\qquad H_{1,I(i)} = -\frac{\sqrt{4\pi}\, i e \hbar^2 c^2}{\sqrt{8h\nu\cdot m_\pi c^2\cdot E^L_{(i)}}}\int (u_I^L)^*\, e^{\frac{i}{\hbar}\boldsymbol{k}\cdot\boldsymbol{r}^{(0T)}}\,\nabla_e\, u^L_{(i)}\, d\tau^{(0T)}.$$

Because of the large energy and correspondingly small wave length of the photon, the integral in (19) is negligible as a result of interference except when the momentum of the meson in the intermediate state is of about the same magnitude as that of the photon but with opposite direction. For these cases, however, the integral is also negligible on account of the operator ∇_e, proportional to the momentum in the direction of polarization of the photon. In comparison with the contribution from the direct transition, the contribution from process (i) can, therefore, be neglected. The same is true of process (ii).

From (15), (16) and (18) it now follows that[1]

$$(20)\qquad P^{ph}_{PS} = \frac{16\pi^2\hbar^3\nu}{(m_\pi c)^3}\cdot\frac{e^2\,|f_{PS}|^2}{\hbar^2 c^2}\cdot|u_I^L(0)|^2\cdot\frac{1}{n_p}\sum_F |I(\sigma_e T_N)|^2$$

where

$$(21)\qquad I(\sigma_e T_N) = \int \varphi^*_{I,0}\left\{\sum_{j=1}^{n} e^{\frac{i}{\hbar}\boldsymbol{k}\cdot\boldsymbol{r}^{(jT)}}\,\sigma_e^{(j)}\,T_N^{(j)}\right\}\varphi_{F,0}\, d\tau^{(1l)}.$$

With the convention on p. 25 $\sum_F |I(\sigma_e T_N)|^2$ is independent of the direction $\boldsymbol{e}$. Disregarding the factors $|u_I^L(0)|^2$ and $\sum_F |I(\sigma_e T_N)|^2/n_p$, (20) is roughly the probability of radiative capture of a free meson by a free proton. The influence of the COULOMB attraction is given by $|u_I^L(0)|^2$, which is the probability of finding the meson inside the nucleus. $\sum_{F} |I(\sigma_e T_N)|^2 / n_p$ gives the influence of the nuclear interaction and will be discussed later on.

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[1] When comparing (20) with the result obtained by CHANG [3] for the free case note that the force constant f_2 in CHANG's calculations is related to f_{PS} by $|f_2| = \sqrt{2\pi}\,|f_{PS}|$.

Calculations for vector mesons. The calculations for vector mesons are similar to those for pseudoscalar mesons. We shall not, therefore, go into the details but confine ourselves to some few remarks.

It is well-known [5, 20] that the PROCA equations for vector mesons in a COULOMB field of a point charge give physically admissible solutions only for those states in which the orbital angular momentum is a rigorous constant of motion. Thus no admissible solutions exist for *s*-states, and the capture of a meson should consequently be excluded. The infinities at the origin, which are much more grave here than in the case of pseudoscalar mesons, can be attributed to the action of the COULOMB field on the magnetic moment of the meson.

This unsatisfactory state indicates that the treatment must be revised in one direction or another.[1] Assuming a breakdown of the COULOMB law at small distances from the centre of action so that everywhere

$$|eV| \ll m_\pi c^2 \tag{22}$$

with slowly varying V, no difficulties arise and the unrelativistic solution is a good approximation, as can be seen by easy calculations. We shall assume below that the condition (22) is satisfied.

Another point also needs some consideration. Unlike the case with pseudoscalar mesons, the contribution from the two-step transitions (i) and (ii) (p. 22) cannot be neglected. Because of interference phenomena only intermediate states with a meson of momentum $\approx -\boldsymbol{k}$ are of importance. The kinetic energy corresponding to this momentum is large ($\approx$ 60 MeV) and the meson can be represented with fairly good approximation by a plane wave, $e^{\frac{i}{\hbar}\boldsymbol{p}\cdot\boldsymbol{r}^{(0\,T)}}$. Indicating by a dash that the summation has to be carried out with specified polarization of the meson in the intermediate state we find

$$\sum_{(i)}{}' \frac{H_{1,I(i)} H_{1,(i)F}}{E_I - E_{(i)}} \approx A \sum_{\boldsymbol{p}} \int (u_I^L)^* \, e^{\frac{i}{\hbar}(\boldsymbol{k}+\boldsymbol{p})\cdot\boldsymbol{r}^{(0\,T)}} \, d\tau^{(0\,T)} \tag{23}$$

where A is roughly a constant. Putting

$$\boldsymbol{P} = \boldsymbol{k} + \boldsymbol{p} \tag{24}$$

and using the FOURIER expansion

$$u_I^L(\boldsymbol{r}^{(0\,T)}) = \sum_{\boldsymbol{P}} B(\boldsymbol{P}) \, e^{\frac{i}{\hbar}\boldsymbol{P}\cdot\boldsymbol{r}^{(0\,T)}} \tag{25 a}$$

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[1] Because of the possibility of pair creation it cannot, for instance, be quite correct to use the one-particle description. Deviations are especially to be expected at small distances from the centre of action, where the absolute value of the potential energy is comparable with or larger than the rest energy of the meson.

with

$$B(\boldsymbol{P}) = \int u_I^L \, e^{-\frac{i}{\hbar} \boldsymbol{P} \cdot \boldsymbol{r}^{(0\,T)}} \, d\,\tau^{(0\,T)} \tag{25 b}$$

we get

$$\sum_{p} \int (u_I^L)^* \, e^{\frac{i}{\hbar} (\boldsymbol{k}+\boldsymbol{p}) \cdot \boldsymbol{r}^{(0\,T)}} \, d\,\tau^{(0\,T)} = \sum_{P} \int (u_I^L)^* \, e^{\frac{i}{\hbar} \boldsymbol{P} \cdot \boldsymbol{r}^{(0\,T)}} \, d\,\tau^{(0\,T)} = u_I^L(0)^*. \tag{26}$$

An analogous discussion applies to process (ii). Thus $H_{1,\,IF}$ is roughly proportional to $u_I^L(0)^*$, which in our approximation is independent of the polarization of the original meson.

Neglecting the difference between $m_\pi c^2$ and $h\nu$ we find after rather lengthy calculations — similar to those in the free case (cf. [2]) — the following expression for the probability of radiative capture:

$$P_V^{ph} = \frac{4\pi\hbar^2}{m_\pi^2 c} \cdot \frac{e^2}{\hbar c} \cdot |u_I^L(0)|^2 \cdot \frac{1}{3\hbar c\, n_p} \sum_F \{|g_V|^2 \, |I(T_N)|^2 + 2\,|f_V|^2 \, |I(\sigma_e T_N)|^2\}. \tag{27}$$

Here $I(\sigma_e T_N)$ is given by (21), while we have

$$I(T_N) = \int \varphi_{I,0}^* \left\{ \sum_{j=1}^{n} e^{\frac{i}{\hbar} \boldsymbol{k} \cdot \boldsymbol{r}^{(j\,T)}} \, T_N^{(j)} \right\} \varphi_{F,0} \, d\,\tau^{(1\,l)}. \tag{28}$$

It is instructive to compare (27) with the corresponding expression for the *free case*, viz.

$$P_V^{ph} = \frac{4\pi\hbar^2}{m_\pi^2 c} \cdot \frac{e^2}{\hbar c} \cdot \frac{|g_V|^2 + 2\,|f_V|}{3\hbar c}. \tag{29}$$

The influence of the nuclear bindings. In order to get an idea of the influence of the nuclear bindings, we must try to estimate $\sum_F |I(T_N)|^2$ and $\sum_F |I(\sigma_e T_N)|^2$ with $I(T_N)$ and $I(\sigma_e T_N)$ given by (28) and (21).

The functions

$$\zeta_I = \left\{ \sum_{j=1}^{n} e^{-\frac{i}{\hbar} \boldsymbol{k} \cdot \boldsymbol{r}^{(j\,T)}} \, T_N^{(j)*} \right\} \varphi_{I,0} \tag{30 a}$$

and

$$\zeta_{II} = \left\{ \sum_{j=1}^{n} e^{-\frac{i}{\hbar} \boldsymbol{k} \cdot \boldsymbol{r}^{(j\,T)}} \, \sigma_e^{(j)} \, T_N^{(j)*} \right\} \varphi_{I,0} \tag{30 b}$$

can be expanded in terms of the eigenfunctions in zero approximation, $\varphi_{\xi,0}$, satisfying an equation of the type (I: 91) generalized for a n-nucleon system. We thus write

$$(31) \qquad \begin{cases} \zeta_I = \sum\limits_{\xi} a_{\xi} \varphi_{\xi,0} \\ \zeta_{II} = \sum\limits_{\xi} b_{\xi} \varphi_{\xi,0} \end{cases} \quad \text{with} \quad \begin{cases} \int \zeta_I^* \, \zeta_I = \sum a_{\xi}^* \, a_{\xi} \\ \int \zeta_{II}^* \, \zeta_{II} = \sum b_{\xi}^* \, b_{\xi}. \end{cases}$$

As has already been pointed out, the main contribution to the capture probability comes from final states with about the same photon energy.[1] We thus get

$$(32\ \text{a}) \qquad \sum_F |I(T_N)|^2 \approx \sum_F a_F^* \, a_F \approx \int \zeta_I^* \, \zeta_I$$

and

$$(32\ \text{b}) \qquad \sum_F |I(\sigma_e \, T_N)|^2 \approx \int \zeta_{II}^* \, \zeta_{II}.$$

In considering (32) we shall use Wigner's [22; cf. also 11 and 24] classification of nuclear states in supermultiplets, analogous to the multiplets in the case of atomic electrons. As quantum numbers Wigner uses the components S_3 and T_3 of the total spin and total isotopic spin together with

$$(33) \qquad Y_3 = \tfrac{1}{2} \sum_{j=1}^{n} \sigma_3^{(j)} \, \tau_3^{(j)}.$$

A supermultiplet is characterized by the symbol (P, P', P'') with $P \geq P' \geq P''$. P is the highest value of, for instance, T_3 in the supermultiplet, while P' is the maximum value of, say, S_3, if $T_3 = P$; P'' is then the highest value of Y_3 consistent with the choice $T_3 = P$ and $S_3 = P'$.

In Wigner's »1st approximation» the potential energy is independent both of the orientation of the spins and the charge states of the nucleons. The terms within a supermultiplet then all have the same energy. The usefulness of the concept of a supermultiplet is understandable for the following reasons. Firstly, the splitting up of the terms in a supermultiplet, due to the spin dependent forces, is small compared with the magnitude of the distance between different supermultiplets in the »1st approximation». Secondly, the dependence of the energy upon T_3, due to the electrostatic forces, is approximately linear within a T-multiplet and a graph of energy against T_3 shows about the same slope for all T-multiplets. The ground state of a nucleus will, therefore, be essentially a state belonging to the lowest supermultiplet in the »1st approximation», consistent with the value T_3, characterizing the charge state. As the energy of a supermultiplet increases with the magnitude of the numbers P, P', P'', the lowest supermultiplet will, in general, have $P = T_3 = T$. The admixture of states belonging to higher supermultiplets increases, however, with increasing mass number because of the decrease in the magnitude of the energy difference between neighbouring supermultiplets. The following discussion does not, therefore, apply with the same degree of accuracy to heavier as to lighter elements.

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[1] The contribution from highly excited states of the nucleus must be expected to be small on account of interference phenomena in the integrals (21) and (28). The presence of the functions $e^{\frac{i}{\hbar} k \cdot r^{(jT)}}$ in the integrals does not affect the validity of the argument, since the kinetic energy of a nucleon corresponding to the momentum k is only about 10 MeV.

In order to illustrate the discussions on p. 22, we shall estimate for different values of the meson mass.

A. *Small Masses*

With a meson mass $\approx 20\ m_e$ ($m_e =$ electron mass) or so, the wave length of the emitted photon is large compared with the nuclear radii. In the expressions (30) for ζ_I and ζ_{II} we can, therefore, put $e^{-\frac{i}{\hbar} k \cdot r^{(jT)}} = 1$.

With the notation

$$\overline{F} = \int \varphi^*_{I,0}\, F\, \varphi_{I,0} \tag{34}$$

where F operates on $\varphi_{I,0}$, we get

$$\int \zeta^*_I\, \zeta_I = \overline{\sum T^{(j)}_N \cdot \sum T^{(j)*}_N} = \overline{(T_1 - i\,T_2)\,(T_1 + i\,T_2)} = T\,(T+1) - T_3(T_3+1). \tag{35}$$

Hence in our approximation

$$\int \zeta^*_I\, \zeta_I = 0. \tag{36}$$

Exception. Elements of the »proton type» ($T_3 = -\tfrac{1}{2}$; $T = \tfrac{1}{2}$) have by (35)

$$\int \zeta^*_I\, \zeta_I = 1\,.$$

Further we get

$$\begin{aligned}\int \zeta^*_{II}\, \zeta_{II} &= \overline{\sum \sigma^{(j)}_e\, T^{(j)}_N \cdot \sum \sigma^{(j)}_e\, T^{(j)*}_N} = \\ &= \tfrac{1}{4}\, \overline{\sum (\sigma^{(j)}_e\, \tau^{(j)}_1 - i\, \sigma^{(j)}_e\, \tau^{(j)}_2) \cdot \sum (\sigma^{(j)}_e\, \tau^{(j)}_1 + i\, \sigma^{(j)}_e\, \tau^{(j)}_2)} = \\ &= \overline{\left(\tfrac{1}{2} \sum \sigma^{(j)}_e\, \tau^{(j)}_1\right)^2} + \overline{\left(\tfrac{1}{2} \sum \sigma^{(j)}_e\, \tau^{(j)}_2\right)^2} - T_3.\end{aligned} \tag{37}$$

$\tfrac{1}{2} \sum \sigma^{(j)}_e\, \tau^{(j)}_1$ and $\tfrac{1}{2} \sum \sigma^{(j)}_e\, \tau^{(j)}_2$ are infinitesimal rotation operators acting in the four-dimensional unitary space of σ and τ together. Since they have only transition elements within a supermultiplet, the mean values of (37) must be independent of the example chosen.

In the case of the supermultiplet (T_3, P', P'') we can thus choose the simple problem with $n_s = 2\,T_3$ neutrons. Putting (cf. notations of I: 95)

$$\psi = \eta^{(1)}_n\, \eta^{(2)}_n \cdots \eta^{(n_s)}_n \cdot \vartheta \tag{38}$$

with ϑ dependent on spin and space coordinates only, we find directly

$$\int \zeta^*_{II}\, \zeta_{II} = \int \psi^* \left\{ \sum_{j=1}^{n_s} \sigma^{(j)}_e\, T^{(j)}_N \cdot \sum_{j=1}^{n_s} \sigma^{(j)}_e\, T^{(j)*}_N \right\} \psi = 0\,. \tag{39}$$

Exceptions: 1) For elements of »proton type», belonging in our approximation to the multiplet $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$, we get $\int \zeta_{II}^* \zeta_{II} = 1$.

2) For elements of »deuteron type» with $T_3 = 0$ and $n = 4k + 2$, the lowest supermultiplet is (1, 0, 0).[1] The triplet state ($S = 1$, $T = 0$) is assumed to be the lowest one within the multiplet. The matrix elements of (37) are here independent of the direction $\boldsymbol{e}$ as a result of our convention implying an averaging over the possible spin directions of the nucleus (cf. p. 25). Instead, we can average over $\boldsymbol{e}$. Using the matrix elements, given by WIGNER [23] for the multiplet (1, 0, 0), we find $\int \zeta_{II}^* \zeta_{II} = \frac{2}{3}$.[2]

From (20), (27) and (32) it can now be seen that the probability of radiative capture for pseudoscalar or vector mesons is small with the exceptions already mentioned. In the latter cases the probability is, however, also smaller (except for some of the lightest nuclei) than a treatment neglecting the bindings would have given; the expressions (32) would then be equal to n_p. In agreement with earlier arguments, it is, therefore, in the case of small meson masses imperative to take the bindings into account and treat the nucleus as a whole. The close analogy to the selection rules in FERMI's and GAMOW-TELLER's formalism for the β-radioactivity is conspicuous.

B. *Large Masses*

In the case of large meson masses it is no longer legitimate to neglect the exponential functions in (30). Since

$$\sum_1^n T_N^{(j)} T_N^{(j)*} = n_p \tag{40}$$

we find, using the symmetry properties of the wave function $\varphi_{I,0}$

$$\int \zeta_I^* \zeta_I = \overline{\sum_1^n e^{\frac{i}{\hbar}\boldsymbol{k}\cdot\boldsymbol{r}^{(jT)}} T_N^{(j)} \cdot \sum_1^n e^{-\frac{i}{\hbar}\boldsymbol{k}\cdot\boldsymbol{r}^{(jT)}} T_N^{(j)*}} = n_p + n(n-1)\cdot \overline{e^{\frac{i}{\hbar}\boldsymbol{k}\cdot\boldsymbol{r}^{(12)}} T_N^{(1)} T_N^{(2)*}} \tag{41 a}$$

and

$$\int \zeta_{II}^* \zeta_{II} = n_p + n(n-1)\cdot \overline{e^{\frac{i}{\hbar}\boldsymbol{k}\cdot\boldsymbol{r}^{(12)}} \sigma_e^{(1)} \sigma_e^{(2)} T_N^{(1)} T_N^{(2)*}}\,. \tag{41 b}$$

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[1] The multiplet (0, 0, 0) is impossible for $n = 4k + 2$, as P, P' and P'' must always satisfy the relation

$$P + P' + P'' = 2m - \tfrac{1}{2}n$$

where m is an integer (cf. [24] p. 284).

[2] Because of the absence of COULOMB interactions in a deuteron, the validity of the result is for a deuteron — on the assumption of charge independence of the nuclear forces — only subject to the approximation arising from neglecting the exponential functions in (30).

From the above discussions it follows that the second terms on the right-hand sides of the relations (41) are very nearly equal to $-n_p$ for small k — with the exception of elements of »proton» and »deuteron type», where they amount approximately to $-n_p + 1$ and $-n_p + \frac{2}{3}$, respectively. In the case of large k, corresponding to a wave length of nuclear dimension, the exponential function cannot be neglected, but will diminish the magnitude of the terms under consideration by interference. In the limit we must expect the value of the integrals (41) to be very close to n_p. The resulting capture probability is then the same as that obtained by treating the nucleons as free.[1]

Noting that the COMPTON wave length of the meson is approximately equal to the wave length of the emitted light quantum the parallelism to the discussion on p. 22 is evident.

Instead of varying the meson mass we could of course let the mass be fixed and change the kinetic energy. The alterations in the calculations are almost obvious, and expressions of the type (32) will again determine the influence of the bindings, thus leading us to the same conclusions as above.

The use of the α-particle model. In the intermediate case with a meson mass neither sufficiently small or large the estimates cannot be made with the same degree of confidence. The interesting value of the mass ($m_\pi \approx 285\, m_e$) is just of this magnitude. In accordance with earlier considerations we shall for this case use the α-particle model of the nucleus.

Putting

$$\varphi_{I,0} = \varphi_A f_A \tag{42}$$

where φ_A is the antisymmetrical wave function (I: 97) for spin and charge and f_A the error function (I: 107), we find by means of (41) with $n_p = 2$

$$\begin{aligned}\int \zeta_I^* \zeta_I = \int \zeta_{II}^* \zeta_{II} = 2 - 2 \int e^{-\frac{i}{\hbar} \boldsymbol{k} \cdot \boldsymbol{r}^{(12)}} f_A^2\, d\tau^{(1\,l)} = \\ = 2\left(1 - e^{-\frac{k^2}{16\,\mu_\alpha \hbar^2}}\right) \approx \tfrac{2}{3}.\end{aligned} \tag{43}$$

Here the values

$$\frac{\hbar}{k} \approx 1.5 \cdot 10^{-13}\ \text{cm} \quad \text{and} \quad \frac{1}{\sqrt{\mu_\alpha}} \approx 3.7 \cdot 10^{-13}\ \text{cm}$$

from (17) and (I: 110) have been used. Generalizing we get

$$\int \zeta_I^* \zeta_I \approx \int \zeta_{II}^* \zeta_{II} \approx \frac{n_p}{3}. \tag{44}$$

Thus we are inclined to believe that the influence of the nuclear bindings can be accounted for by correcting the probability of radiative capture for free nucleons by a factor of about 1/3. This applies to mesons of low kinetic energy. It is not, however, difficult to see that even in other cases the nuclear bindings will introduce a correction factor roughly amounting to

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[1] If the meson mass is of the same order of magnitude as M or larger, our calculations are no longer applicable. The necessary modifications will not, however, affect our conclusion.

$$1 - e^{-\frac{k^2}{16\,\mu_\alpha\,\hbar^2}} \qquad (k c \approx \text{meson energy}). \tag{45}$$

Thus for example, if the kinetic energy of the meson is equal to its rest energy, the correction factor is about 0.8.

The capture of scalar and pseudovector mesons. In the case of pseudovector mesons the coupling H_{AMN}, responsible for the direct transitions, contains only terms proportional to $\bar{S}_0$ or $\overline{M}_{ij}$ (I: 85). These operators introduce the components $\varphi_{I,(j)}$ and $\varphi_{F,(j)}$ (cf. I: 32), which are relativistic of the first order. The contribution from the direct transitions is, therefore, much smaller than in the case of vector mesons, where $\bar{S}_0$ and $\overline{M}_{ij}$ are replaced by S_0 and M_{ij}. The indirect transitions give, on the other hand, as can easily be seen, a contribution of about the same magnitude as for vector mesons. The probability of radiative capture is, therefore, nevertheless of roughly the same order of magnitude for both kinds of mesons.

The replacement of $\overline{\boldsymbol{S}}$ by $\boldsymbol{S}$ in H_{AMN} (cf. 8 c) for scalar mesons brings about a considerable reduction in the capture probability compared with that for pseudoscalar mesons. Using the approximation

$$\varphi_{I,(j)} = \frac{1}{2\,M\,c}\left(\boldsymbol{\sigma}^{(j)} \cdot \boldsymbol{p}^{(j)}\right) \varphi_{I,0} \tag{46}$$

and the corresponding one for $\varphi_{F,(j)}$ — implying that the motion of the centre of gravity is neglected — the capture probability can be obtained from that for pseudoscalar mesons (20)[1] by substituting $\sum\limits_F |I'|^2$ for $\sum\limits_F |I(\sigma_e T_N)|^2$ with

$$I' = \frac{1}{2\,M\,c}\int \varphi_{I,0}^{*}\left\{\sum_{j=1}^{n}\left[\left(\boldsymbol{\sigma}^{(j)}\cdot\boldsymbol{p}^{(j)}\right)\sigma_e^{(j)}\, e^{\frac{i}{\hbar}\boldsymbol{k}\cdot\boldsymbol{r}^{(jT)}} + e^{\frac{i}{\hbar}\boldsymbol{k}\cdot\boldsymbol{r}^{(jT)}}\sigma_e^{(j)}\left(\boldsymbol{\sigma}^{(j)}\cdot\boldsymbol{p}^{(j)}\right)\right] T_N^{(j)}\right\}\varphi_{F,0}\, d\tau^{(1l)}. \tag{47}$$

Introducing the function

$$\zeta_S = \left\{\sum_{j=1}^{n}\left[\left(\boldsymbol{\sigma}^{(j)}\cdot\boldsymbol{p}^{(j)}\right)\sigma_e^{(j)}\, e^{-\frac{i}{\hbar}\boldsymbol{k}\cdot\boldsymbol{r}^{(jT)}} + e^{-\frac{i}{\hbar}\boldsymbol{k}\cdot\boldsymbol{r}^{(jT)}}\sigma_e^{(j)}\left(\boldsymbol{\sigma}^{(j)}\cdot\boldsymbol{p}^{(j)}\right)\right] T_N^{(j)*}\right\}\varphi_{I,0} \tag{48}$$

we get (cf. p. 28, 29)

$$\begin{aligned}\sum_F |I'|^2 &\approx \frac{1}{4\,M^2\,c^2}\int \zeta_S^{*}\,\zeta_S \approx \frac{1}{4\,M^2\,c^2}\int \varphi_{I,0}^{*}\left\{\sum\left(k^2 + \tfrac{4}{3}\,p^{(j)^2}\right)\tfrac{1}{2}\left(1-\tau_3^{(j)}\right)\right\}\varphi_{I,0} \approx \\ &\approx \frac{n_p}{4\,M^2\,c^2}\left(k^2 + \tfrac{4}{3}\,\overline{p^{(j)^2}}\right) \approx \frac{n_p}{80}\end{aligned} \tag{49}$$

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[1] The two-step transitions (i) and (ii) (cf. p. 22) also give negligible contributions for scalar mesons. As the relativistic components of the wave functions must be taken into account, a closer examination of the process in which the photon is emitted by the capturing nucleon is, however, necessary. It can be shown that the contribution from this process may also be neglected.

where we have put $\overline{p^{(j)^2}}/2M \approx 10$ MeV. The probability of radiative capture for scalar mesons is thus smaller than that for pseudoscalar mesons by a factor 25 or so.

It is seen from (49) that the internal motion of the nucleus is of importance. The treatment of the nucleons as free and at rest gives a capture probability about one half of that obtained if the bindings are taken into account.

Comparisons

We are now ready to compare the probability of process (1) ($\pi^- + P \rightarrow N$) with the probability of process (2) ($\pi^- + P \rightarrow N + h\nu$). Choosing the case of vector mesons, we get from earlier discussions and with the help of (I: 114) a probability of the former process given by

$$P_V = \alpha\beta\,|\,u_I^L(0)\,|^2 \frac{|\,g_V\,|^2 + 2\,|\,f_V\,|^2}{3\hbar c}\, 7\cdot 10^{-27}\cdot c \ \text{sec}^{-1}. \tag{50}$$

Here $|\,u_I^L(0)\,|^2$, being the square of the amplitude of the wave function of the meson inside the nucleus, shows the influence of the electrostatic interaction between the meson and the nucleons. The factor α, roughly satisfying $\frac{1}{20} \leq \alpha \leq \frac{1}{6}$ (cf. p. 22), is the correction necessary when changing the mass from $\approx 200\, m_e$ to $285\, m_e$, while β is the correction arising from the possible errors in the calculations in Part I.

The probability of radiative capture is, on the other hand, from (27), (32) and (44) given by

$$P_V^{ph} = |\,u_I^L(0)\,|^2 \frac{|\,g_V\,|^2 + 2\,|\,f_V\,|^2}{3\hbar c}\, 6\cdot 10^{-28}\cdot c \ \text{sec}^{-1}. \tag{51}$$

As has already been pointed out, the great frequency of stars, where negative π-mesons end in photographic emulsions, indicates that process (1) dominates for slow mesons. Since it seems as if (51) could be used with fair confidence, it must, therefore, be considered as a lower limit for the probability of process (1), the calculation of which is afflicted by much more grave sources of error.

In order to attain consistency it seems thus necessary to assume that $\beta > 1$. However, in Part I reasons were given for believing that the most important sources of error in the calculations will cause a reduction in the capture probability. This view is further supported by the discussion in this paper about the contribution from other final states than that containing a triton and an emitted neutron. If the experiments are correctly interpreted, a deficiency in the theory seems, therefore, to be indicated.

Nothing much is changed by applying the above discussion to pseudoscalar or pseudovector mesons. The capture probability seems to be roughly the same as for vector mesons. In the scalar case the calculations indicate a considerably smaller probability of both capture with and capture without photoemission.

Appendix

We shall treat here the case where the capturing of the meson by an α-particle leads to its disintegration into four free nucleons, three neutrons and one proton, without simultaneous emission of any quanta (photon, neutral meson). The interaction between the nucleons in the final state is neglected and we may represent the wave functions of the initial and final states by

$$\left\{\begin{aligned} \psi_I &= \varphi_I\, e^{\frac{i}{\hbar} p_I^{(T)}\cdot r^{(T)}} \\ \psi_F &= \omega_F\, e^{\frac{i}{\hbar}\sum\limits_{k=1}^{4} p_F^{(k)}\cdot r^{(k)}} \end{aligned}\right. \tag{52}$$

where φ_I and ω_F describe the spin and charge states and φ_I also depends on the relative coordinates of the nucleons.

The calculations are very similar for different meson theories with quite analogous results, and we may therefore confine ourselves to the treatment of *vector mesons* with the further limitation $f_v = 0$. The transition element corresponding to (I: 80) is then given by

$$H_{1,IF,(\varrho)} = \frac{\hbar g_V}{Mc}\sqrt{\frac{2\pi}{\mu}}\int \psi_{I,0}^{*}\left\{\sum_{k=1}^{4}(p_F^{(k)}\cdot e^{(\varrho)})\, T_N^{(k)}\right\}\psi_{F,0}\, d\tau^{(1)}\cdots d\tau^{(4)} \tag{53}$$

where the index 0 refers to the unrelativistic approximation of the wave functions.

In conformity with the calculations in Part I, we shall use the representation

$$\varphi_{I,0} = \varphi_A f_A \tag{54}$$

where φ_A is the antisymmetrical spin and charge function (I: 97) and f_A the symmetrical error function (I: 107). We shall further choose

$$\omega_{F,0} = \chi_+^{(1)}\chi_+^{(2)}\chi_-^{(3)}\chi_-^{(4)}\cdot\eta_n^{(1)}\eta_n^{(2)}\eta_n^{(3)}\eta_p^{(4)}. \tag{55}$$

Permutations of spin and charge indices will give other possible choices. We have in all 24 different possibilities, each giving the same contribution to the capture probability.

Using (54) and (55) and putting $p_I^{(T)} = 0$ we get after some easy calculations

$$H_{1,IF,(\varrho)} = \frac{\hbar g_V}{2Mc}\sqrt{\frac{\pi}{3\mu}}\{(p_F^{(1)} - p_F^{(2)})\cdot e^{(\varrho)}\}\int f_A\, e^{\frac{i}{\hbar}\sum\limits_{1}^{4} p_F^{(k)}\cdot r^{(k}}\, d\tau^{(1)}\cdots d\tau^{(4)}. \tag{56}$$

Introducing coordinates of the centre of gravity and relative coordinates, the integration over the former gives as usual the conservation law, i. e.

$$\sum_{1}^{4} p_F^{(k)} = 0. \tag{57}$$

Performing the remaining integrations we obtain

$$(58) \qquad \int f_A \, e^{\frac{i}{\hbar}\sum\limits_1^4 \boldsymbol{p}_F^{(k)} \cdot \boldsymbol{r}^{(k)}} \, d\tau^{(1)} \cdots d\tau^{(4)} = \frac{c_\alpha}{64}\left(\frac{\pi}{\mu_\alpha}\right)^{\frac{9}{2}} e^{-\frac{M E_{\mathrm{kin}}}{8 \mu_\alpha \hbar^2}}$$

with

$$(59) \qquad E_{\mathrm{kin}} = \frac{1}{2M}\sum_1^4 p_F^{(k)^2}.$$

Taking into consideration the 24 possible ways of choosing the spin and charge dependence we now get the total capture probability per proton[1]

$$(60) \qquad P'_V = 24\,\frac{\pi}{\hbar}\sum_F |H_{1,\,I\,F,\,(\varrho)}|^2 = \frac{\pi^2 \hbar\, c_\alpha^2}{2^{11}\mu\, M^2 c^2}\left(\frac{\pi}{\mu_\alpha}\right)^9 |g_V|^2\, e^{-\frac{M E_{\mathrm{kin}}}{4\mu_\alpha \hbar^2}} \cdot \sum_F \{(\boldsymbol{p}_F^{(1)} - \boldsymbol{p}_F^{(2)}) \cdot \boldsymbol{e}^{(\varrho)}\}^2$$

where the summation has to be carried out over all final states per unit energy interval excluding differences in spin and charge dependence. We get further

$$(61) \qquad \sum_F \{(\boldsymbol{p}_F^{(1)} - \boldsymbol{p}_F^{(2)}) \cdot \boldsymbol{e}^{(\varrho)}\}^2 = \tfrac{1}{3}\sum_F (\boldsymbol{p}_F^{(1)} - \boldsymbol{p}_F^{(2)})^2.$$

Neglecting $\sum\limits_F \boldsymbol{p}_F^{(1)} \cdot \boldsymbol{p}_F^{(2)}$, which is comparatively small, and using the symmetry with regard to the four nucleons, (61) is simplified to

$$(62) \qquad \sum_F \{(\boldsymbol{p}_F^{(1)} - \boldsymbol{p}_F^{(2)}) \cdot \boldsymbol{e}^{(\varrho)}\}^2 = \tfrac{1}{3}\sum_F M E_{\mathrm{kin}} = \tfrac{1}{3} M E_{\mathrm{kin}}\, \varrho_{E_F}$$

where ϱ_{E_F} is the number of final states per unit energy interval.

The calculation of ϱ_{E_F} can be performed in the following way. Using the conservation law (57) we get

$$(63) \qquad E_F = E_{\mathrm{kin}} + 4Mc^2 = \frac{1}{2M}\left\{\sum_1^3 p_F^{(l)^2} + \left(\sum_1^3 \boldsymbol{p}_F^{(l)}\right)^2\right\} + 4Mc^2.$$

Putting

$$(64) \qquad \gamma^2 = 2 M E_{\mathrm{kin}}$$

and introducing

$$(65) \qquad I_\gamma = \int d\boldsymbol{p}_F^{(1)}\, d\boldsymbol{p}_F^{(2)}\, d\boldsymbol{p}_F^{(3)}$$

where the integration is extended over the volume $\sum\limits_1^3 p_F^{(l)^2} + \left(\sum\limits_1^3 \boldsymbol{p}_F^{(l)}\right)^2 \leq \gamma^2$, we may then write

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[1] P'_V refers to a free meson. The influence of the COULOMB interaction is approximately accounted for by multiplying P'_V by the square of the wave amplitude of the meson inside the nucleus.

$$\varrho_{E_F} = \frac{M}{\gamma h^9} \frac{d I_\gamma}{d\gamma}. \tag{66}$$

With the new variables

$$\begin{cases} \hat{\boldsymbol{p}}^{(1)} = \boldsymbol{p}_F^{(1)} + \tfrac{1}{2} (\boldsymbol{p}_F^{(2)} + \boldsymbol{p}_F^{(3)}) \\ \hat{\boldsymbol{p}}^{(2)} = \dfrac{\sqrt{3}}{2} (\boldsymbol{p}_F^{(2)} + \tfrac{1}{3} \boldsymbol{p}_F^{(3)}) \\ \hat{\boldsymbol{p}}^{(3)} = \sqrt{\dfrac{2}{3}} \boldsymbol{p}_F^{(3)}. \end{cases} \tag{67}$$

I_γ is transformed into

$$2\sqrt{2} \int d\hat{\boldsymbol{p}}^{(1)} \, d\hat{\boldsymbol{p}}^{(2)} \, d\hat{\boldsymbol{p}}^{(3)} \tag{68}$$

the integration being now extended over $\sum_1^3 \hat{p}^{(l)^2} \leq \gamma^2/2$. Apart from the factor $2\sqrt{2}$, (68) represents the volume of a nine-dimensional sphere with radius $\gamma/\sqrt{2}$. The volume must be of the form $c_9 \, (\gamma/\sqrt{2})^9$ and the value of c_9 may be taken from the relation

$$1 = \frac{1}{(2\pi)^{\frac{9}{2}}} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{-\frac{1}{2} \sum_1^9 \xi_i^2} d\xi_1 \cdots d\xi_9 = \frac{9 c_9}{(2\pi)^{\frac{9}{2}}} \int_0^{\infty} e^{-\frac{R^2}{2}} R^8 \, dR = \frac{9 \cdot 7 \cdot 5 \cdot 3}{2 \cdot (2\pi)^4} c_9 \tag{69}$$

giving

$$c_9 = \frac{32 \pi^4}{945}. \tag{70}$$

Hence

$$I_\gamma = \frac{4 \pi^4}{945} \gamma^9 \tag{71}$$

and using (66)

$$\varrho_{E_F} = \frac{4 \pi^4}{105} \frac{M}{h^9} (2 M E_{\text{kin}})^{\frac{7}{2}}. \tag{72}$$

From (60), (62) and (72) it now follows that

$$P'_V = \frac{\pi^4}{2^{12} \cdot 105} \left(\frac{\pi}{\mu_\alpha}\right)^9 \frac{c_\alpha^2}{\mu M c \hbar^7} (2 M E_{\text{kin}})^{\frac{9}{2}} e^{-\frac{M E_{\text{kin}}}{4 \mu_\alpha \hbar^2}} \frac{|g_V|^2}{3 \hbar c}. \tag{73}$$

If the f_V-dependent part of the interaction is taken into account, $|g_V|^2$ has to be replaced by $|g_V|^2 + 2|f_V|^2$, as is easily verified.

P'_V can now be compared with the probability P_V (I: 114) of a capture resulting in a triton nucleus and an emitted neutron. The evaluation refers to a meson with a rest energy of 100 MeV. Consequently we have

$$E_{\text{kin}} \approx 70 \text{ MeV}.$$

With the values of c_α and μ_α given by (I: 108 a) and (I: 110), we ultimately obtain

$$P'_V \approx 7\,P_V. \tag{74}$$

SUMMARY

The results of a preceding paper [1] are applied to the process $\pi^- + P \rightarrow N$. The modifications necessary when passing over from a meson mass of about 200 m_e to $m_\pi \approx 285\ m_e$ are discussed. The α-particle model of the nucleus is used and special attention is payed to final states implying a total disruption of the α-particle, as these will gain in importance with increasing mass of the meson. The calculations refer to the capture of slow mesons.

Because of the great uncertainty of the estimate of the probability of the process, a comparison with the probability of radiative capture ($\pi^- + P \rightarrow N + h\nu$) is made. Since the frequency of stars at the end of π^--meson tracks indicates that the former process dominates, a lower limit of this is thus assumed to be obtained. The limit seems, however, to be somewhat too high to be in good agreement with the estimated probability of capture without photoemission.

It is further shown that the influence of the nuclear bindings cannot be neglected when evaluating the probability of radiative capture, but can — except for the case of scalar mesons — be roughly accounted for by the factor $1 - e^{-\frac{k^2}{16\mu_\alpha \hbar^2}}$, where k is the momentum of the emitted photon and $1/\sqrt{\mu_\alpha} \approx \approx 3.7 \cdot 10^{-13}$ cm. In the case of the capture of a slow π-meson the factor is about 1/3.

General arguments are given for the use of the α-particle model when a slow meson is captured or produced.

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REFERENCES: [1] **Bruno**, Ark. f. Mat., Astr. o. Fys. *36 A*, N:o 8, 1948. — [2] **Chang**, Proc. Camb. Phil. Soc. *36*, 34, 1940. — [3] ——, K. Danske Vidensk. Selsk. Mat.-fys. Medd. *19*, No. 10, 1942. — [4] **Conversi, Pancini** and **Piccioni,** Phys. Rev. *71*, 209 and 577, 1947. — [5] **Corben** and **Schwinger,** Phys. Rev. *58*, 953, 1940. — [6] **Dyson,** Phys. Rev. *73*, 617, 1948. — [7] **Fermi, Teller and Weisskopf,** Phys. Rev. *71*, 314, 1947. — [8] **Fröhlich, Huby, Kolodziejski** and **Rosenberg,** Nature *162*, 450, 1948. — [9] **Gardner** and **Lattes,** Science March 12, 1948. — [10] **Heitler,** Proc. Roy. Soc. *166*, 529, 1938. — [11] **Hund,** ZS. f. Phys. *105*, 202, 1937. — [12] **Kissinger** and **Cooper,** Phys. Rev. *74*, 349, 1948. — [13] **Kobayasi** and **Okayama**, Proc. Phys.-Math. Soc. Japan *21*, 1, 1939. — [14] **Kramers**, Proc. Amst. Akad. Sci. *40*, 814, 1937. — [15] **Lodge**, Nature *161*, 809, 1948. — [16] **Occhialini** and **Powell**, Nature *161*, 551, 1948. — [17] ——, Nature *162*, 168, 1948. — [18] **Snyder** and **Weinberg**, Phys. Rev. *57*, 307, 1940. — [19] **Sommerfeld**, Wellenmechanik, New York, p. 216. — [20] **Tamm**, Phys. Rev. *58*, 952, 1940. — [21] **Ticho**, Phys. Rev. *74*, 492 and 1337, 1948. — [22] **Wigner**, Phys. Rev. *51*, 106 and 947, 1937. — [23] ——, Phys. Rev. *56*, 519, 1939. — [24] **Wigner** and **Feenberg**, Rep. Progr. Phys. *8*, 274, 1941.

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